Substituting this, $m = 1$, and $\delta_1 = \delta_C$ into Equation 9.18, we have

$$n_{cr} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_C}}$$

$$= \frac{1}{2\pi}\left[\frac{9.81(256)(200\times10^9)(0.3\times10^{-6})}{3(400)(1)^3}\right]^{1/2}$$

$$= 56.40 \text{ cps} = 3384 \text{ rpm}$$

9.8 Mounting Parts

Mounting parts, such as keys, pins, screws, ring, collars, and splines, are usually used on shafts to attach the hub of rotating members such as gears, pulleys, sprockets, cams, and flywheels. Note that the portion of the mounted members in contact with the shaft is the *hub*. The hub is attached to the shaft in variety of ways, using one of the foregoing mounting elements. Each mounting configuration has its own advantages and disadvantages. Tables of dimensions for the mounting parts may be found in engineering handbooks and manufacturer's catalogs.

9.8.1 Keys

A *key* enables the transmission of torque from the shaft to the hub. Numerous kinds of keys are used to meet various design requirements. They are standardized as to size and shape in several styles. Figure 9.10 illustrates a variety of keys. The grooves in the shaft and hub into which the key fits form the *keyways* or key seats. The square, flat type of keys is most common in machine construction.

The *gib-head key* is tapered so that, when firmly driven, it prevents relative axial motion. Another advantage is that the hub position can be adjusted for the best location. A tapered key may have no head or a gib head (as in Figure 9.10d) to facilitate removal. The *Woodruff key* is semicircular in plan and of constant width (w). It is utilized widely in the automotive and machine tool industries. Woodruff keys yield better concentricity after assembly of the hub and shafts. They are self-aligning and accordingly preferred for tapered shafts.

9.8.2 Pins

A pin is employed for axial positioning and the transfer of relatively light torque or axial load (or both) to the hub. Some types of shaft pins are the straight round pin, tapered round pin, and roll pin (Figure 9.11). The so-called roll pin is a split-tubular spring pin. It has sufficient flexibility to accommodate itself to small amounts of misalignment and variations in hole diameters, so it does not come loose under vibrating loads.

9.8.3 Screws

Very wide keys can be held in place with countersunk flat head or *cap screws* if the shaft is not weakened. In addition to a key or pin, *setscrews* are often employed to keep the hub from shifting axially on the shaft. For light service, the rotation between shaft and hub also

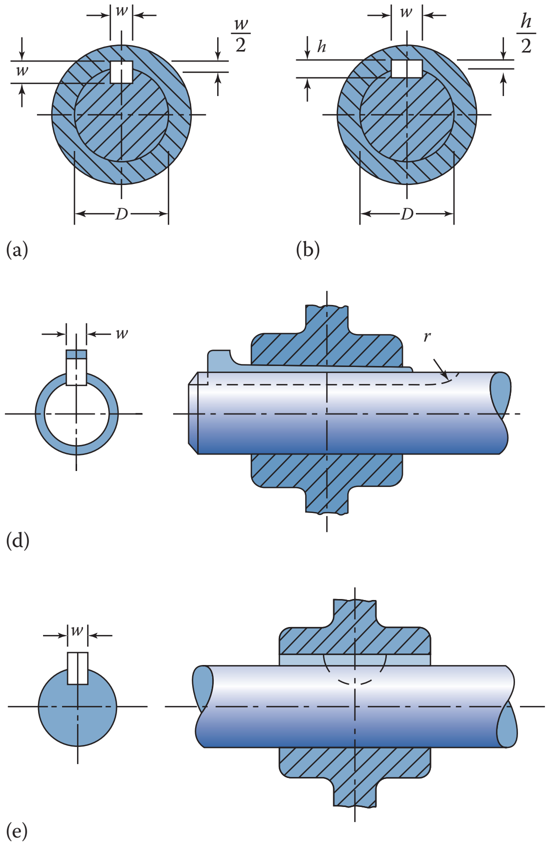

FIGURE 9.10
Common types of shaft keys: (a) square key ($w \approx D/4$), (b) flat key ($w \approx D/4$, $h \approx 3w/4$), (c) round key (often tapered), (d) gib-head key, and (e) Woodruff key.

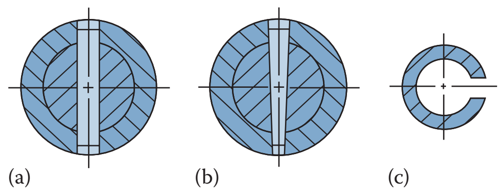

FIGURE 9.11
Some types of pins: (a) straight round pin, (b) tapered round pin, and (c) cross section of a split-tubular pin or so-called roll pin.

may be prevented by setscrews alone. Setscrews are sometimes used in combination with keys. Various types of screws and standardized screw threads are discussed in Chapter 15.

9.8.4 Rings and Collars

Retaining rings, commonly referred to as *snap rings*, are available in numerous varieties and require that a small groove of specific dimensions be machined in the shaft. Keys, pins, and snap rings can be avoided by the use of *clamp collars* that squeeze the outside

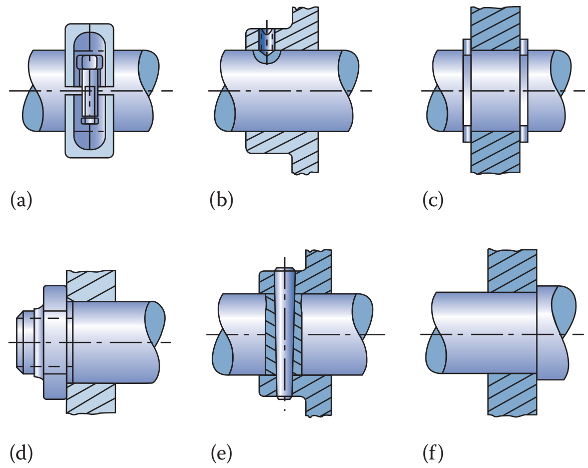

FIGURE 9.12
Various means of securing hubs for axial motion: (a) clamp collar, (b) setscrew, (c) snap rings, (d) nut, (e) tapered pin, and (f) interference fit.

diameter of the shaft with high pressure to clamp something to it. The hub bore and clamp collar have a matching slight taper. The clamp collar with axial slits is forced into the space between hub and shaft by tightening the bolts.

9.8.5 Methods of Axially Positioning of Hubs

Figure 9.12 shows common methods of axially positioning and retaining hubs into shafts. Axial loads acting on shafts or members mounted on the shaft are transmitted as follows: light loads by clamp joints, setscrews, snap rings, and tapered keys (Figure 9.10); medium loads by nuts, pins, and clamp joints; and heavy loads by press or shrink fits. Interference fits are also used to position and retain bearings into hubs.

9.9 Stresses in Keys

The distribution of the force on the surfaces of a key is very complicated. Obviously, it depends on the fit of the key in the grooves of the shaft and hub. The stress varies nonuniformly along the key length; it is highest near the ends.

Owing to many uncertainties, an exact stress analysis cannot be made. However, it is commonly assumed in practice that a key is fitted as depicted in Figure 9.13. This implies that the entire torque T is carried by a tangential force F located at the shaft surface and uniformly distributed along the full length of the key:

$$T = Fr \tag{9.22}$$

where r is the shaft radius.

Shear and compressive or bearing stresses are calculated for the keys from force F, using a sufficiently large factor of safety. For steady loads, a *factor of safety* of 2 is commonly applied. On the other hand, for minor to high shock loads, a factor of safety of 2.5–4.5 should be used.

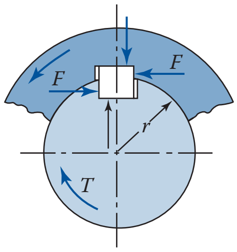

FIGURE 9.13
Forces on a key tightly fitted at the top and bottom.

For keyways, the concentration of stress depends on the values of the fillet radius at the ends and along the bottom of the keyways. For end-milled key seats in shafts under either bending or torsion loading, the theoretical *stress-concentration factors* range from 2 to about 4, contingent on the ratio r of r/D [9]. The quantity r represents the fillet radius (see Figure 9.10d) and D is the shaft diameter. The approximate values of the fatigue stress-concentration factor range between 1.3 and 2 [4].

Example 9.7: Design of a Shaft Key

A shaft of diameter D rotates at 600 rpm and transmits 100 hp through a gear. A square key of width w is to be used (Figure 9.10a) as a mounting part. Determine the required length of the key.

Given: $D = 50$ mm, $w = 12$ mm.

Design Decisions: The shaft and key will be made of AISI 1035 cold-drawn steel having the same tensile and compressive yield strength and that yield strength in shear is S_{ys}, $= S_y/2$. The transmitted power produces intermittent minor shocks and a factor of safety of $n = 2.5$ is used.

Solution

From Table B.3, for AISI 1035 CD steel, we find $S_y = 460$ MPa. Through the use of Equation 1.16,

$$T = \frac{7121(100)}{600} = 1.187\ \text{kN}\cdot\text{m}$$

The force F at the surface of the shaft (Figure 9.13) is

$$F = \frac{T}{r} = \frac{1.187}{0.025} = 47.48\ \text{kN}$$

On the basis of shear stress in the key,

$$\frac{S_y}{2n} = \frac{F}{wL} \quad \text{or} \quad L = \frac{2Fn}{S_y w} \tag{9.23}$$

Substitution of the given numerical values yields

$$L = \frac{2(47,480)(2.5)}{460(10^6)(0.012)} = 43\ \text{mm}$$

Based on compression or bearing on the key or shaft (Figure 9.10a),

$$\frac{S_{yc}}{n} = \frac{F}{(w/2)L} \quad \text{or} \quad L = \frac{2Fn}{S_{yc}w} \tag{9.24}$$

Inasmuch as $S_y = S_{yc}$, this also results in $L = 43$ mm.

9.10 Splines

When axial movement between the shaft and hub is required, relative rotation is prevented by means of splines machined on the shaft and into the hub. For example, splines are used to connect the transmission output shaft to the drive shaft in automobiles, where the suspension movement causes axial motion between the components. Splines are essentially *built-in keys*. They can transform more torque than can be handled by keys. There are two forms of splines (Figure 9.14): straight or square tooth splines and involute tooth splines. The former is relatively simple and employed in some machine tools, automatic equipment, and so on. The latter has an involute curve in its outline, which is in widespread use on gears. The involute tooth has less stress concentration than the square tooth and, hence, is stronger. Also easier to cut and fit, the involute splines are becoming the prominent spline form.

Formulas for the dimensions of splines are based on the nominal shaft diameter. Figure 9.14a shows the standard SAE 6 and 10 straight spline fittings. Note that the values of root diameter d, width w, and depth h of the internal spline are based on the nominal shaft diameter D or about the root diameter of the external spline. According to the SAE, the torque capacity (in lb·in.) of *straight-sided splines* with sliding is

$$T = pnr_mhL_c \tag{9.25}$$

where

T = the theoretical torque capacity
n = the number of splines
$r_m = (D + d)/4$, mean or pitch radius (see Figure 9.14)
h = the depth of the spline
L_c = the length of the spline contact
p = the spline pressure

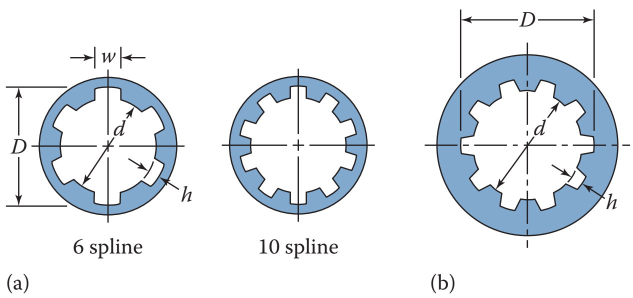

FIGURE 9.14
Some common types of splines: (a) straight-sided and (b) involute.